Question Bank

Department of Mathematics

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B.Sc. (Hons.) Mathematics

Paper: C3 Real Analysis (Semester II, CBCS)

Multiple Choice Questions

1. Infimum of the set $(0,\infty)$
   (a) is a non-negative number.
   (b) is a positive number.
   (c) does not exist.
   (d) none of these.
2. Which of the following is not true for a set in **R**?
   (a) A set may not have an infimum in **R**.
   (b) Infimum of a set may not belong to the set.
   (c) Infimum and supremum of a set may be equal.
   (d) Supremum of a bounded below set always exists in **R**.
3. Which algebraic property is not true for the set of real numbers **R**?
   (a) For all $a \neq 0$, $b \in$ **R** such that a.b = 1 implies b = 1/ a.
   (b) a. (1/ a) = 1 for all $a \neq 0$.
   (c) $\sqrt{a^2} = a$ for all $a \in$ **R**.
   (d) If a.b > 0 then either a > 0 and b > 0 or a < 0 and b < 0.
4. Which of the following is true for a bounded below subset S of the set of real number **R**?
   (a) Sup (cS) = c Sup (S) for $c \in$ **R**.
   (b) – Inf (S) = Sup (–S).
   (c) $a \in S$ such that $a^2 > 0$ implies $a > 0$.
   (d) none of these.
5. For $\epsilon = \frac{1}{8}$, the least natural number n such that terms of the sequence $\left(\frac{1}{n}\right) \in \epsilon-$ neighbourhood of 0 is
   (a) 9
   (b) 8
   (c) ∞

(d) none of these.

6. Which of the following is not true for the following sequences?

(a) $\lim_{n\to\infty}\left(\frac{n}{n^2+1}\right) = 0.$

(b) $\lim_{n\to\infty}\left(\frac{2n}{n+1}\right) = 2.$

(c) $\lim_{n\to\infty}\left(\frac{n^2}{n!}\right) = 0.$

(d) $\lim_{n\to\infty}\left(\sqrt{n^2+1} - n\right)$ does not exist.

7. Which of the following sequences is convergent?

(a) (n)

(b) $((-1)^n)$

(c) $\left(\frac{\sin n}{n}\right)$

(d) none of these.

8. Which of the following is true?

(a) Every decreasing sequence of real number is convergent.

(b) Every monotone sequence is convergent.

(c) Constant sequence is not convergent.

(d) $\left(\frac{1}{\sqrt{n}}\right)$ is convergent.

9. Let $a_n = \frac{4^{3n}}{3^{4n}}$. Then the sequence (a_n)

(a) is unbounded.

(b) is bounded but not convergent.

(c) converges to 0.

(d) converges to 1.

10. Sup $\left\{\left(\sqrt{n+1} - \sqrt{n}\right)\right\}$ and Inf $\left\{\left(\sqrt{n+1} - \sqrt{n}\right)\right\}$ are

(a) $\sqrt{2} + 1$ and 0 respectively.

(b) $\frac{1}{\sqrt{2}+1}$ and 0 respectively.

(c) both equal to 0.

(d) both equal to $\frac{1}{\sqrt{2}+1}$.

11. Which of the following sets is countable?

(a) The set [0,1].

(b) The set **R** of all real numbers.

(c) The set **Q** of all rational numbers.

(d) None of these.

12. The sequence $\left(\frac{\sin(\frac{n\pi}{2})}{n}\right)$

(a) is convergent.

(b) is divergent.

(c) is convergent to 0.

(d) is convergent to 1.

13. If the sequence is convergent then

(a) it has two limits.

(b) it is bounded.

(c) it is bounded above but may not be bounded below.

(d) it is bounded below but may not be bounded above.

14. If the sequence is increasing, then it

(a) converges to its supremum.

(b) diverges.

(c) may converge to its supremum.

(d) is bounded.

15. If $S = \{ \frac{1}{n} - \frac{1}{m} : n, m \in \mathbf{N}\}$ where **N** is the set of natural number. Then infimum and supremum of S respectively are

(a) -1 and 1.

(b) 0 and 1.

(c) 0 and 0.

(d) cannot be determined.

16. Which of the following is not true?

(a) The set [0,1] is a finite set.

(b) The set **R** of all real numbers is uncountable.

(c) The set **Q** of all rational numbers is countable.

(d) None of these.

17. Which of the following series converges?

(a) $\sum_{n=1}^{\infty} 1$.

(b) $\sum_{n=1}^{\infty} (-1)^{n+1}$.

(c) $\sum_{n=1}^{\infty} (-1)^n \left(\frac{1}{n}\right)$.

(d) $\sum_{n=1}^{\infty} \left(\frac{1}{n}\right)$.

18. Which of the following is true?

(a) $\sum_{n=1}^{\infty} (-1)^n \left(\frac{1}{2n-1}\right)$ is not conditionally convergent.

(b) $\sum_{n=1}^{\infty}(-1)^n e^{-n}$ is absolutely convergent.

(c) $\sum_{n=1}^{\infty}(-1)^n \left(\frac{\cos n\pi}{n}\right)$ is convergent.

(d) $\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}\right)$ is not convergent.

19. If **S** and **T** are two subsets of the set of real numbers **R** such that $\mathbf{S} \subseteq \mathbf{T}$ then

(a) inf **T** $\leq$ inf **S**.

(b) inf **T** $\geq$ inf **S**.

(c) sup **T** $\leq$ sup **S.**

(d) none of these.

20. Let **S** be a nonempty set, and let f and g be defined on **S** and have bounded ranges in the set of real numbers **R**. Then

(a) $\sup\{ f(x) + g(x) : x \in \mathbf{S}\} \geq \sup\{ f(x) : x \in \mathbf{S}\} + \sup\{ g(x) : x \in \mathbf{S}\}$.

(b) $\inf\{ f(x) + g(x) : x \in \mathbf{S}\} \geq \inf\{ f(x) : x \in \mathbf{S}\} + \inf\{ g(x) : x \in \mathbf{S}\}$.

(c) $\sup\{ a + f(x) : x \in \mathbf{S}\} \neq a + \sup\{ f(x) : x \in \mathbf{S}\}$.

(d) none of these.

Short Answer Type Questions

1. Define Supremum and Infimum of a set. Also give examples.
2. State and prove Archimedean Property in **R**.
3. Let S be a nonempty subset of **R** that is bounded below. Prove that

$$\text{Inf } S = -\text{Sup}\{-s : s \in S\}.$$

4. Use the definition of limit to establish the following limit:

$$\lim_{n\to\infty}\left(\frac{3n+1}{2n+5}\right) = \frac{3}{2}.$$

5. If $0 < a < b$, determine, $\lim_{n\to\infty}\left(\frac{a^{n+1}+b^{n+1}}{a^n+b^n}\right)$.
6. Use Squeeze Theorem to determine the limit of the following sequences:

(i) $\left(n^{\frac{1}{n^2}}\right)$

(ii) $\left((n!)^{\frac{1}{n^2}}\right)$

(iii) $\left(\frac{\text{Sin } n}{n}\right)$

(iv) $(a^n + b^n)^{\frac{1}{n}}$, where $0 < a < b$.

7. Define a Cauchy Sequence and also state the Cauchy Convergence Criterion.
8. If the series $\sum x_n$ is convergent, then show that $\lim_{n\to\infty}(x_n) = 0$.
9. State Alternating Series test.
10. Define Absolute Convergence and Conditional Convergence in a series.

Does Absolute convergence imply convergence? Is the converse true?

Long Answer Type Questions

1. (i) State Completeness Property of **R.**

   (ii) Let S be any nonempty subset of **R** that is bounded above, and let a be any real number then show that $\sup(a + S) = a + \sup S$.

2. (i) State the Order Properties of **R.**

   (ii) Show that there exists a positive real number x such that $x^2 = 2$.

3. If $X = (x_n)$ converges to x and $Z = (z_n)$ is a sequence of non-zero real numbers that converges to z and if $z \neq 0$, then show that the quotient sequence $\frac{X}{Z}$ converges to $\frac{x}{z}$.

4. Let (x_n) be a sequence of positive real numbers such that $L = \lim_{n\to\infty}\left(\frac{x_{n+1}}{x_n}\right)$ exists. Show that if L < 1, then (x_n) converges and $\lim_{n\to\infty}(x_n) = 0$. Hence, find

$$\lim_{n\to\infty}\left({}^{n}/{}_{b^n}\right), \text{ where } b > 1.$$

5. Let (x_n) be a sequence of positive real numbers such that $\lim_{n\to\infty}\left(\frac{x_{n+1}}{x_n}\right) = L > 1$. Show that X is not a bounded sequence and hence is not convergent.

6. Let $X = (x_n)$ be the sequence of real numbers defined by $x_1 = 1,\ x_{n+1} = \sqrt{2 + x_n}$ for $n \in$ **N.** Show that (x_n) converges and find the limit.

7. Show that the series $\sum \frac{1}{n^p}$ is convergent if and only if p >1.

8. State and prove Limit Comparison test.

9. State and prove Ratio test.

10. Test for convergence the following series:

    (i) $\sum_{n=1}^{\infty} \frac{1}{2n+1}$

    (ii) $\sum_{n=1}^{\infty} \frac{1}{(2n+1)^2}$

    (iii) $\sum_{n=1}^{\infty} \frac{2^{n+1}}{n^2 3^{n-1}}$

    (iv) $\sum_{n=0}^{\infty} \frac{n+1}{2^n}$

    (v) $\sum_{n=1}^{\infty} n e^{-n^2}$

    (vi) $\sum_{n=1}^{\infty} (-1)^{n+1}\left(\frac{1}{n}\right)$

Paper: C4 Differential Equations (Semester II, CBCS)

Multiple Choice Questions

1. The order of the differential equation whose general solution is given by $y = (c_1 + c_2)\cos(x + c_3) - c_4 e^{x+c_5}$, where c_1, c_2, c_3, c_4, c_5 are arbitrary constants, is

   (a) 5

   (b) 4

   (c) 3

   (d) 2

2. Which of the following is not an integrating factor of $xdy - ydx = 0$?

   (a) $\frac{1}{x^2}$

   (b) $\frac{1}{x^2+y^2}$

   (c) $\frac{1}{xy}$

   (d) $\frac{x}{y}$

3. The differential equation $(y - 2x^3)dx - x(1 - xy)dy = 0$ becomes exact on multiplication with which one of the following?

   (a) x

   (b) x^2

   (c) $\frac{1}{x^2}$

   (d) $\frac{1}{x}$

4. What is the solution of the equation $x\frac{dy}{dx} + \frac{y^2}{x} = y$?

   (a) $\log\left(\frac{y}{x}\right) - \left(\frac{1}{x}\right) = c$

   (b) $\log(x) - \left(\frac{x}{y}\right) = c$

   (c) $\log\left(\frac{x}{y}\right) - \left(\frac{1}{x}\right) = c$

   (d) $\log(x) + \left(\frac{x}{y}\right) = c$

5. What is the solution of the differential equation?

   $\frac{dy}{dx} = (4x + y + 1)^2$?

   (a) $4x + y + 1 = 2\tan(2x + y + c)$

   (b) $4x + y + 1 = 2\tan(x + 2y + c)$

(c) $4x + y + 1 = 2\tan(2y + c)$

(d) $4x + y + 1 = 2\tan(2x + c)$

Short Answer Type Questions

1. In Lotka-Volterra Model of Predator-Prey population what happens to the Prey in the absence of any Predator?
2. What are the limitations of Competing Species Model?
3. In Epidemic Model for Influenza, what is Latent period and Incubation period? Which period is longer?
4. Define Random fire and Aimed fire in a Battle Model.
5. Write down the differential equation describing the concentration of pollution in a lake. How does this change the model if only unpolluted water flows into the lake?
6. Solve the equation
$$(3x^2 + 4xy)dx + (2x^2 + 2y)dy = 0.$$
7. Suppose $n \neq 0, 1$. Then the transformation $v = y^{1-n}$ reduces the Bernoulli equation
$$\frac{dy}{dx} + P(x)y = Q(x)y^n$$
to linear equation in v. Prove it.
8. Solve the following equation
$$\cos(y)\frac{dy}{dx} + \frac{1}{x}\sin(y) = 1.$$
9. Find a general solution of
$$y''' + y' - 10y = 0.$$
10. A spherical tank of radius 4 ft. is full of gasoline when a circular bottom hole with radius 1 in. is opened. How long will be required for all the gasoline to drain from the tank?

Long Answer Type Questions

1. A population of rabbits $X(t)$ is preyed upon by a population of foxes $Y(t)$. A model for this population interaction is the pair of differential equations
$$\frac{dX}{dt} = -aXY, \quad \frac{dY}{dt} = bXY - cY$$
where a, b, c are positive constants.
   (i) Find the relationship between the density of foxes $Y(t)$ and the density of rabbits $X(t)$.
   (ii) Is it possible for the foxes to completely wipe out rabbit population? Give reasons.
2. In a fish farm, fish are harvested at a constant rate of 2100 fish per week. The per capita death rate of the fish is 0.2 fish per day per fish and the per capita birth rate is 0.7 fish per day per fish.

(i) Draw the compartmental diagram. Write down the word equation describing the rate of change of fish population. Hence obtain the differential equation for the number of fish.

(ii) If the fish population at a given time is 24000, give an estimate of the number of fish born in one week.

3. Consider the aimed fire Battle Model with differential equations

$$\frac{dR}{dt} = -a_1 B, \quad \frac{dB}{dt} = -a_2 R$$

where a_1 and a_2 are positive constants.

(i) Find the exact solution for R and B where R denotes the number of soldiers in red army and B denotes the number of soldiers in blue army.

(ii) Find the arbitrary constants of integration by solving the simultaneous equations for $R(0) = r_0$ and $B(0) = b_0$, when $t = 0$.

4. Consider a disease where all those infected remain contagious for life. A model describing this is given by the differential equations

$$\frac{dS}{dt} = -\beta SI, \quad \frac{dI}{dt} = \beta SI$$

where β is a positive constant.

(i) Obtain and sketch the phase plane curves. Determine the direction of travel along the trajectories.

(ii) Using this model, is it possible for all susceptible to be infected?

5. Solve the initial-value problem:

$$(3x - y - 6)dx + (x + y + 2)dy = 0, \quad y(2) = -2.$$

6. (i) Prove that if f and g are two different solutions of

$$\frac{dy}{dx} + P(x)y = Q(x), \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(A)}$$

then $f - g$ is a solution of the equation

$$\frac{dy}{dx} + P(x)y = 0.$$

(ii) Thus show that if f and g are two different solutions of Equation (A) and c is an arbitrary constant, then

$$c(f - g) + f$$

is a one-parameter family of solution of (A).

7. A motorboat starts from rest (initial velocity $v(0) = v_0 = 0$). Its motor provides a constant acceleration of $4\, ft/s^2$, but water resistance causes a deceleration of $\frac{v^2}{400} ft/s^2$. Find v when $t = 10\ s$, also find the limiting velocity as $t \to +\infty$ (that is, the maximum possible speed of the boat).

8. Solve the initial-value problem:

$$y'' + 2y' + y = 0;$$

$$y(0) = 5, \quad y'(0) = -3.$$

9. Find general solution (for $x > 0$) of the following ordinary differential equation (Euler's equation):

$$x^2y'' + xy' + 9y = 0.$$

10. Use the method of variation of parameters to find a particular solution of the following differential equation:

$$y'' - 2y' - 8y = 3e^{-2x}.$$

Paper: C8 Partial Differential Equations (Semester IV, CBCS)

Multiple Choice Questions

1. The general solution of the linear partial differential equation

$$a(x, y, u)p + b(x, y, u)q = c(x, y, u) \text{ is}$$

(a) $f(\phi, \psi) = 1$

(b) $f(\phi, \psi) = -1$

(c) $f(\phi, \psi) = 0$

(d) None of these

2. Characteristics for the equation $(y^2z)p + (zx)q = y^2$ are

(a) $\frac{dx}{y^2z} = \frac{dy}{zx} = \frac{dz}{y^2}$

(b) $\frac{dx}{x^2} = \frac{dy}{y^2} = \frac{dz}{zx}$

(c) $\frac{dx}{y^2} = \frac{dy}{x^2} = \frac{dz}{zx}$

(d) $\frac{dx}{zx} = \frac{dy}{y^2z} = \frac{dz}{y^2}$

3. The partial differential equation $u_{xx}+x^2u_{yy} = 0$ is classified as

(a) elliptic

(b) parabolic

(c) hyperbolic

(d) none of the above

4. The solution of the below initial- value problem is

$$u_{tt} = c^2u_{xx}, x\epsilon R, t > 0,$$

$$u(x, 0) = \sin x, u_t(x, 0) = \cos x$$

(a) $\sin x \cos ct - \frac{1}{c} \cos x \sin ct$

(b) $\sin x \cos ct + \cos x \sin ct$

(c) $\sin x \cos ct$

(d) $\sin x \cos ct + \frac{1}{c} \cos x \ \sin ct$

5. The following is true for the partial differential equation used in nonlinear mechanics known as Korteweg-de Vries equation.

$$w_t + w_{xxx} - 6\, w\, w_x = 0$$

(a) linear; 3rd order

(b) nonlinear; 3rd order

(c) linear; 1st order

(d) nonlinear 1st order

Short Answer Type Questions

1. Find the solution of the Cauchy problem

$$3u_x + 2u_y = 0, \text{ with } u(x,0) = \sin x$$

2. Find the general solution of the equation

$$u_x + 2xy^2 u_y = 0$$

3. Reduce the following equation to canonical form

$$u_{xx} + 5u_{xy} + 4u_{yy} + 7u_y = \sin x$$

4. Determine the solution of the initial value problem

$$u_{tt} - c^2 u_{xx} = 0, u(x,0) = x^3, u_t(x,0) = x$$

5. Find the solution of the following problem

$$u_{tt} = c^2 u_{xx} = 0, 0 < x < \pi, t > 0,$$
$$u(x,0) = \sin x\,, u_t(x,0) = x^2 - \pi x, 0 \le x \le \pi,$$
$$u(0,t) = u(\pi,t) = 0, t > 0$$

Long Answer Type Questions

1. Apply $\sqrt{u} = v$ and $v(x,y) = f(x) + g(y)$ to solve the equation

$$x^4 u_x^2 + y^2 u_y^2 = 4u$$

2. Show that the equation of motion of a long string is

$$u_{tt} = c^2 u_{xx} - g$$

where g is the gravitational acceleration.

3. Transform the equation

$$u_{xy} + y u_{yy} + \sin(x+y) = 0$$

into the canonical form. Use the canonical form to find the general solution.

4. Determine the solution of the Goursat problem

$$u_{tt} = c^2 u_{xx},$$
$$u(x,t) = f(x), on\ x - ct = 0$$
$$u(x,t) = g(x), on\ t = t(x)$$

where $f(0) = g(0)$.

5. Solve the problem

$$u_t - ku_{xx} = 0, 0 < x < 1, t > 0$$
$$u(x,0) = x(1-x), 0 \leq x \leq 1,$$
$$u(0,t) = t, u(1,t) = sin\,t\,, t \geq 0$$

Paper: C9 Riemann Integration & Series of Functions(Semester IV, CBCS)

Multiple Choice Questions (More than one statement can also be correct)

1. Approximation of the definite integral $\int_0^8 2x\,dx$ with the Riemann sum by dividing [0,8] into 4 equal subintervals and taking midpoint of each interval is
   (a) 186
   (b) 168
   (c) 167
   (d) None of these
2. Upper Darboux integral for the function $f(x) = \begin{cases} 1, & x \in \mathbb{Q} \\ 0, & x \notin \mathbb{Q} \end{cases}$ on the interval $[0,b]$ is
   (a) $U(f) < \frac{b^2}{2}$
   (b) $U(f) = \frac{b^2}{2}$
   (c) $U(f) > \frac{b^2}{2}$
   (d) None of these
3. Let $f_n : [0,1] \to [0,1]$ be a sequence of differentiable functions. Assume that (f_n) converges uniformly on $[0,1]$ to a function f. Then
   (a) f is differentiable and Riemann integrable on [0,1]
   (b) f is uniformly continuous and Riemann integrable on [0,1]
   (c) f is continuous, f need not be differentiable on $(0,1)$ and need not be Riemann integrable on [0,1]
   (d) f need not be uniformly continuous on [0,1]
4. Consider the power series $\sum_{n=1}^{\infty} a_n x^n$ where $a_0 = 0$ and $a_n = \frac{\sin(n!)}{n!}$ for $n \geq 1$. Let R be the radius of convergence of the power series. Then
   (a) $R \geq 1$

(b) $R \geq 2\pi$

(c) $R \leq 4\pi$

(d) All are true

5. For any positive integer n, let $f_n : [0,1] \to \mathbb{R}$ be defined by $f_n(x) = \frac{x}{nx+1}$ for $x \in [0,1]$. Then

(a) The sequence (f_n) converges uniformly on $[0,1]$

(b) The sequence $(f_n{}')$ of derivatives of (f_n) converges uniformly on $[0,1]$

(c) The sequence $(\int_0^1 f_n(x)\, dx)$ is convergent

(d) The sequence $(\int_0^1 f_n{}'(x)\, dx)$ is convergent

6. Let $f_n(x) = x^n$ for $x \in [0,1]$ and $n \in \mathbb{N}$. Then

(a) $\lim_{n\to\infty} f_n(x)$ exists $\forall\, x \in [0,1]$

(b) $\lim_{n\to\infty} f_n(x)$ defines a continuous function on $[0,1]$

(c) (f_n) converges uniformly on $[0,1]$

(d) $\lim_{n\to\infty} f_n(x) = 0\ \forall\, x \in [0,1]$

7. Let $\{a_n : n \geq 1\}$ be a sequence of real numbers such that $\sum_{n=1}^{\infty} a_n$ is convergent and $\sum_{n=1}^{\infty} |a_n|$ is divergent. Let R be the radius of convergence of the power series $\sum_{n=1}^{\infty} a_n\, x^n$. Then we can conclude that:

(a) $0 < R < 1$

(b) $R = 1$

(c) $1 < R < \infty$

(d) $R = \infty$

8. Let f be a monotonically increasing function from $[0,1]$ to $[0,1]$. Which of the following statements is/are true?

(a) f must be continuous at all points in $[0,1]$

(b) f must be differentiable at all points in $[0,1]$

(c) f must be Riemann Integrable

(d) f must be bounded

9. Let $f: [0,1] \to \mathbb{R}$ be the function given by $(x) = \begin{cases} 1, & if\ 0 \leq x < 0.5 \\ 2, & if\ 0.5 \leq x < 0.7 \\ 3, & if\ 0.7 \leq x \leq 1 \end{cases}$.

Then

(a) f is not Riemann integrable

(b) f is Riemann integrable and $\int_0^1 f(x)\, dx = 2$

(c) f is Riemann integrable and $\int_0^1 f(x)\, dx = 2.1$

(d) None of the above

10. Which of the following statement(s) is/are not true?
    (a) Every Riemann integrable function is bounded
    (b) Every monotone function on $[a, b]$ is Riemann integrable
    (c) Every continuous function is Riemann integrable
    (d) If a function f on [0,1] be defined by

$$f(x) = \begin{cases} k, & if\ x = \frac{1}{k}\ ,\ k \in \mathbb{N} \\ 0, & elsewhere \end{cases}$$

is not Riemann integrable.

Short Answer Type Questions

1. Find the pointwise limit of the (f_n) defined by $f_n(x) = \frac{x^n}{1+x^n},\ x \in [0,2]$.
2. Define radius of convergence of a power series and find the radius of convergence of the series $\sum_{n=2}^{\infty} \frac{1}{\ln(n)}$.
3. Show that $\sum_{n=1}^{\infty} \frac{1}{x^2+n^2},\ x \in \mathbb{R}$ is uniformly convergent on $\mathbb{R}$.
4. Let $(s_n) = (0,1,2,1,0,1,2,1,0,1,2,1, \dots)$ and $(t_n) = (2,1,1,0,2,1,1,0,2,1,1,0, \dots)$. Then find
   (i) $\liminf\limits_{n\to\infty} s_n$ and $\limsup\limits_{n\to\infty} t_n$
   (ii) $\liminf\limits_{n\to\infty} s_n + \liminf\limits_{n\to\infty} t_n$
   (iii) $\text{Liminf}\limits_{n\to\infty}(s_n + t_n)$
5. Define uniform convergence of sequence of functions and give one example of uniformly convergent sequence of functions.
6. Find the upper Darboux integral for $f(x) = x^3$ on the interval $[-b, b],\ b > 0$.
7. Show that if f is integrable on $[a, b]$ then f^3 is integrable on $[a, b]$.
8. Let $f \geq 0$ and integrable on [a,b] . Is $\sqrt{f}$ integrable on $[a, b]$?

Long Answer Type Questions

1. A function f on $[a, b]$ is called a *step function* if there exists a partition $P = \{a = t_0 < t_1 < \cdots < t_n = b\}$ of $[a, b]$ such that f is constant on each interval (t_{k-1}, t_k), say $f(x) = c_k$ for $x \in (t_{k-1}, t_k)$. Show that a step function f is integrable and evaluate it.
2. Let $f(x) = \sin\frac{1}{x}$ for $x \neq 0$ and $f(0) = 0$. Show f is integrable on $[-1, 1]$.
3. Let (f_n) be a sequence of integrable functions on $[a, b]$, and suppose $f_n \to f$ uniformly on $[a, b]$. Prove that f is integrable on $[a, b]$ and $\int_a^b f = \lim\limits_{n\to\infty} \int_a^b f_n$.
4. Let $f(x) = x\ sgn(sin\ \frac{1}{x})$ for $x \neq 0$ and $f(0) = 0$.

(i) Show that f is not piecewise continuous on $[-1,1]$.

(ii) Show that f is not piecewise monotonic on $[-1,1]$.

(iii) Show that f is integrable on $[-1,1]$.

5. Suppose f and g are continuous functions on $[a,b]$ such that $\int_a^b f = \int_a^b g$. Prove that $\exists\ x$ in (a,b) such that $f(x) = g(x)$.

6. Show that if $a > 0$, then (f_n) defined as $f_n(x) = tan^{-1}(nx)$ converges uniformly to $f(x) = \frac{\pi}{2} sgn(x)$ on the interval $[a,\infty)$ but is not uniformly convergent on $(0,\infty)$.

7. Let $f(x) = \sum_{n==1}^{\infty} a_n x^n$ be a power series with finite positive radius of convergence R. Then f is differentiable on $(-R,R)$ and $f'(x) = \sum_{n==1}^{\infty} n a_n x^{n-1}$ for $|x| < R$.

8. Let (s_n) be a sequence of real numbers. Then

(i) If $\lim_{n\to\infty} s_n$ is defined (as a real number, +∞, -∞), then

$$\liminf_{n\to\infty} s_n = \lim_{n\to\infty} s_n = \limsup_{n\to\infty} s_n$$

(ii) If $\limsup_{n\to\infty} s_n = \liminf_{n\to\infty} s_n$, then $\lim_{n\to\infty} s_n$ is defined and

$$\lim_{n\to\infty} s_n = \limsup_{n\to\infty} s_n = \liminf_{n\to\infty} s_n$$

Paper: C10 Ring Theory & Linear Algebra-I (Semester IV, CBCS)

Multiple Answer Type Questions

Pick the correct statement(s) in following. More than one statement can also be true in following.

1. $\mathbb{Z}$, the Set of integers is

(a) Integral Domain

(b) Principal Ideal Domain

(c) Division Ring

(d) Commutative Ring

2. If A and B are ideals in a Commutative Ring with Unity R, then

(a) A ∩ B is an ideal

(b) A ∪ B is an ideal

(c) AB is an ideal

(d) All of the above

3. Let $\mathbb{F}$ be a finite field with cardinality 32. Then *Char*($\mathbb{F}$) can be

(a) 6

(b) 32

(c) 2

(d) None of the these

4. Let $\mathbb{F}$ be an infinite field. Then

   (a) $\mathbb{F}$ is a Multiplicative Group

   (b) $\mathbb{F}$ is an Additive Group

   (c) $\mathbb{F}\sim\{0\}$ is an Additive Group

   (d) $\mathbb{F}\sim\{0\}$ is a Multiplicative Group

5. $\mathbb{Z}_8$, the ring of integers modulo 8 has

   (a) No units

   (b) No Zero Divisors

   (c) No Nilpotents

   (d) No Idempotents

6. Let V be an infinite vector space over a field $\mathbb{F}$ and β be the basis of V. Then

   (a) β is linearly dependent

   (b) β is a generating set for V

   (c) β is linearly independent

   (d) None of the above

7. Pick basis for $\mathbb{R}^2$ over $\mathbb{R}$

   (a) {(1, -2), (3, 0)}

   (b) {(0, -2), (-11, 0)}

   (c) {(11, -2), (0, 0)}

   (d) {(5,-2), (3, 0)}

8. Pick basis for $P_2(\mathbb{R})$ over $\mathbb{R}$

   (a) $\{1, 0, x^2\}$

   (b) $\{1, x, x^2\}$

   (c) $\{1, 1+x, 1+x^2\}$

   (d) $\{1, 1- x, x^2\}$

9. If β is a basis of a vector space V. Then

   (a) β is a subset of V

   (b) β is a subspace of V

   (c) β is a unique subset of V

   (d) β is a unique subspace of V

10. If W and U be two subspaces of a vector space V over a field F. Then
    (a) W+U forms a subspace
    (b) W ∩ U forms a subspace
    (c) W ∪ U forms a subspace
    (d) All of the above

Short Answer Type Questions

1. Give an example of a non-commutative Division Ring.
2. Give an example of a commutative ring without unity in which a maximal ideal is not a prime ideal.
3. Prove that $\langle x \rangle$ is a prime ideal in $\mathbb{Z}[x]$ but not a maximal ideal in $\mathbb{Z}[x]$.
4. Verify that $\mathbb{R}^3$ is a vector space over $\mathbb{R}$.
5. Prove that $\frac{\mathbb{R}[x]}{<x>}$ is an Integral Domain.

Long Answer Type Questions

1. Prove that the direct sum of finitely many Rings remains a Ring under component wise addition and multiplication. What can you say about direct product of finitely many Integral Domains? Justify.
2. Prove that a field cannot have a composite cardinality.
3. State and Prove *Second Isomorphism Theorem* for Rings.
4. State and Prove *Third Isomorphism Theorem* for Rings.
5. Let T be a linear transformation from V to W then prove that T(V) is a subspace of W.

Paper: SEC-2: Computer Algebra Systems and Related Softwares (Semester IV, CBCS)

Short Answer Type Questions

1. Define a function $f(x) = x^3 + \sin x$ in Maple and evaluate $f(\pi/2)$.
2. Define a function $f(x) = x^3 + \cos x$ in Maxima. Find the differentiation of $f(x)$ with respect to x.
3. Compute 3^6 mod 7 using Mathematica.
4. Let $A = \begin{bmatrix} 1 & -3 & -3 \\ -1 & 1 & 2 \\ 0 & -2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -3 & 1 \\ 3 & 0 & 2 \\ 2 & 1 & 2 \end{bmatrix}$

   Compute C = AB using MATLAB.

5. Put the list of values 7, 5, 9, 2, 1, 8, 4, 2, 4, 8 into a variable a.

   (i) Find the mean, median, and sample standard deviation of a.

   (ii) Sort the array a.

Long Answer Type Questions

1. Put the following values into a file (using Notepad or some other suitable editor), and read the file into a variable b using R software.

   9 9 5 6 9 2 5 1 9 9 1 4 8 10 4 5 4 1 8 2 5 8 2 9 4 1 2 6 3 2 9 7 6 4 6

   (i) Generate a five number summary of b.

   (ii) Create a box plot of b.

   (iii) Create a stem and leaf plot of b.

2. Let $M = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$.

   (i) Find $M^2, M^3, \ldots, M^{10}$ (Using Maple).

   (ii) Do your answers suggest a way to compute Fibonacci numbers? Find the 100th Fibonacci number.

3. Graph each of the functions using Maxima. Experiment with different domains or viewpoints to display the best images.

   (i) $f(x) = \frac{x}{1+x^2}$

   (ii) $y = x \sin \frac{1}{x}$

4. Let $f(x) = \frac{x}{1+x^2}$. (Using Mathematica)

   (i) Find $f'(x)$ and $f''(x)$.

   (ii) Find $f'(-1)$ and $f'(0)$.

   (iii) Find $f''(0)$ and $f''(1)$.

5. Solve the system of equation using MATLAB

$$-2x - 4z = -8$$
$$-4x - 3y = -20$$
$$-2y = -8$$

Paper: Complex Analysis (Semester VI)

Multiple Choice Questions

1. The harmonic conjugate of $u(x, y) = y^3 - 3x^2y$ is
   (a) $-y^3 + 3x^2y + c, (where\ c\ is\ a\ constant)$
   (b) $y^3 - 3xy^2 + c$
   (c) $x^3 - 3x^2y + c$
   (d) $x^3 - 3xy^2 + c$
2. The domain of convergence of the series $\sum_{n=0}^{\infty} n^2 \left(\frac{z^2+1}{1+i}\right)^n$ is
   (a) $|z + 1| < 2$
   (b) $|z^2 + 1| < \sqrt{2}$
   (c) $|z^2 + 1| < 2$
   (d) $|z + 1| < \sqrt{2}$
3. At z = 0 , the function $f(z) = \frac{1-cosz}{z}$ has
   (a) a simple pole
   (b) an essential singularity
   (c) a removable singularity
   (d) a non-isolated singularity
4. Radius of convergence of the power series $1 + \frac{z^2}{a^2} + \frac{z^4}{a^4} + \cdots, a \neq 0$
   (a) $|a|^{-1}$
   (b) $|a|$
   (c) 0
   (d) none
5. The function $f(z) = conjugate(z)$ maps the third quadrant onto
   (a) itself
   (b) lower half plane
   (c) upper half plane
   (d) second quadrant
6. By stereographic projection with the south pole at the origin (0,0,0) the point (1,0,1) goes to the complex number
   (a) $z = i$
   (b) $z = 1 + i$
   (c) $z = 1$
   (d) none

7. If $f = u + iv$ is an analytic function, it satisfies the Cauchy-Riemann equations
   (a) $u_x = v_y and\ u_y = v_x$
   (b) $u_x = v_x\ and\ u_y = v_y$
   (c) $u_x = v_x and\ u_y = \ -v_y$
   (d) $u_x = v_y and\ u_y = -v_x$
8. Let $f(z) = \begin{cases} \frac{z}{|z|}\ if\ z \neq 0 \\ 0\ if\ z = 0 \end{cases}$. Then
   (a) f is discontinuous at 0
   (b) f is not analytic at 0 but is continuous at 0
   (c) f is differentiable at 0
   (d) f is not differentiable at 0 but it is continuous at 0
9. Let f be an entire function. If $Re(f)$ is bounded then
   (a) $Im(f)$ is constant
   (b) f is constant
   (c) f = 0
   (d) All are correct
10. If $1 + i\sqrt{3} = r\ e^{i\theta}$ then
   (a) $r = 2\ and\ \theta = \frac{\pi}{3}$
   (b) $r = 2\ and\ \theta = \frac{\pi}{4}$
   (c) $r = 1\ and\ \theta = \frac{\pi}{3}$
   (d) $r = 1\ and\ \theta = \frac{\pi}{4}$

Short Answer Type Questions

1. Solve the following equations in polar form and locate the roots in the complex plane:
   (i) $z^6 = 1.$
   (ii) $z^4 = \ -1 + \ \sqrt{3}i$
2. Find the radius of convergence of
   (i) $\Sigma\ \frac{(-1)^n z^n}{n!}$
   (ii) $\Sigma\ \frac{n! z^n}{n^n}$
   (iii) $\Sigma\ n^p c_n z^n\ if\ \Sigma c_n z^n has\ radius\ of\ convergence\ R.$
3. Find all analytic functions $f = u + i\ v$ with $u(x, y) = x^2 - y^2$.
4. Find the power series expansion of $f(z) = z^2$ around $z = 2$.

5. Classify the singularities of

   (i) $\frac{1}{z^4+z^2}$

   (ii) $cot\, z$

   (iii) $\frac{e^{1/z^2}}{z-1}$

Long Answer Type Questions

1. State and prove *Weistrass M-Test*.
2. Suppose f_x and f_y exist in a neighborhood of z. Then if f_x and f_y are continuous at z and $f_y = i\, f_x$ there, f is differentiable at z.
3. If f is entire and if C is a (smooth) closed curve ,

$$\oint f(z)dz = 0.$$

4. State and prove *Morera's* theorem.
5. State and prove *Cauchy Integral Formula*.

Paper: Algebra-V (Semester VI)

Multiple Choice Questions

1. The number of permissible cycle types in S_5 is
   (a) 7
   (b) 4
   (c) 5
   (d) None
2. The number of 3-sylow subgroups of group of order 25 is
   (a) 1
   (b) 3
   (c) 0
   (d) 5
3. The group $Z_m \times Z_n$ is cyclic if
   (a) m n = 1
   (b) m + n = 1
   (c) g.c.d(m, n) = 1
   (d) l.c.m(m,n) = 1
4. The number of conjugate classes of Q_8 is
   (a) 8
   (b) 4

(c) 7

(d) 5

5. The number of groups of order 49 is

(a) 4

(b) 1

(c) 7

(d) 2

6. The number of elements of order 4 in $Z_2 \times Z_4$ is

(a) 8

(b) 4

(c) 6

(d) 2

7. The number of simple groups of order 60 is

(a) 1

(b) 10

(c) 60

(d) 6

8. The number of conjugacy classes of elements of order 4 in S_3 is

(a) 6

(b) 1

(c) 0

(d) 2

9. What is the largest order of any element in U(900):

(a) 900

(b) 40

(c) 60

(d) 100

10. If G is an abelian group of order 20, then the number of possible isomorphism classes of G is

(a) 2

(b) 6

(c) 5

(d) 20

11. The number of sylow 3-subgroups of A_4 is

(a) 1

(b) 24

(c) 4

(d) 5

12. If G is an abelian group of order 60, then number of sylow 5-subgroups of G is

(a) 10

(b) 9

(c) 60

(d) 6

Short Answer Type Questions

1. The set {1, 9, 16, 22, 29, 53, 74, 79, 81} is a group under multiplication modulo 91. Determine the isomorphism class of this group.
2. Let

$$A = \begin{pmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{pmatrix}$$

Find an orthogonal matrix P and a diagonal matrix D such that $P^t AP = D$.

3. If $\alpha_1 = (1\ 5)(3\ 7\ 2)(10\ 6\ 8\ 11)$ and $\alpha_2 = \alpha_1^3$, then check whether α_1 and α_2 are conjugate or not and if they are, find μ such that

$$\mu\, \alpha_1\, \mu^{-1} = \alpha_2.$$

4. Find all conjugate classes of D_6.
5. Find all subgroups of order 3 in $Z_9 \times Z_3$.
6. Show that A_5 is simple.

Long Answer Type Questions

1. Find all conjugate classes of A_4, D_6 and S_4.
2. Prove that if |G|=385, then show that
   (i) a sylow 7-subgroup is normal
   (ii) a sylow 11-subgroup is normal
   (iii) Z(G) contains a sylow 11-subgroup.
3. The differentiation operator on the space of polynomials of degree less than or equal to 3 is represented by

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

What is the Jordan form of this matrix?

4. Show that a group of order 12 is not simple.

5. Let T be a linear operator on R^3 which is represented in the standard ordered basis by the matrix:

$$A = \begin{pmatrix} 5 & 6 & 6 \\ 1 & 4 & 2 \\ 3 & 6 & 4 \end{pmatrix}$$

Find its rational form and hence find an invertible 3×3 real matrix P such that $P^{-1}AP$ is in rational form.

Paper: Discrete Mathematics (Semester VI)

Multiple Choice Questions

1. A complete graph with more than two vertices is not bipartite.
   (a) True
   (b) False
2. Isomorphism is an equivalence relation on the set of all graphs.
   (a) True
   (b) False
3. If **A** is the adjacency matrix of the graph K_5, then the (2,4) entry of A^2 is
   (a) 2
   (b) 3
   (c) 4
   (d) 5
4. If a graph **G** has adjacency matrix $\begin{matrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{matrix}$, how many walks from v back to itself that include two edges.
   (a) 1
   (b) 2
   (c) 3
   (d) 4
5. $K_{8,10}$ is Hamiltonian.
   (a) True
   (b) False
6. If a graph **G** contains a cycle that does not pass through all the vertices of **G,** then **G** is not Hamiltonian.
   (a) True
   (b) False

7. A graph with more than one component cannot be Eulerian.
   (a) True
   (b) False
8. A pseudograph that possesses an Eulerian trail has exactly two odd vertices.
   (a) True
   (b) False
9. If graphs G_1 and G_2 are isomorphic and pictures are drawn for each of these graphs then the pictures must look
   (a) Exactly same
   (b) different
   (c) similar
   (d) could be any kind
10. It is not possible for a graph to have degree of sequence 6, 5, 4, 3, 2.
   (a) True
   (b) False

Short Answer Type Questions

1. Find the CN form of the function f = (x (y' + z)) + z'. Hence find its DN form.
2. Determine the symbolic representation of the circuit given by: $(x_1' x_2)' + x_3$.
3. Show that every distributive lattice is modular. Is the converse true?
4. State and prove connecting lemma.
5. Draw the Hasse diagram of the following ordered subsets of $(N_0, \prec)$:
   (i) {1, 2, 3, 4, 6, 12}
   (ii) {1, 2, 3, 12, 18, 0)

Long Answer Type Questions

1. Prove that two finite ordered sets P and Q are order isomorphic if and only if they can be drawn with identical diagrams.
2. Show that every finite lattice is complete.
3. Let L be a lattice. Show that the following are equivalent:

$$(D)\ \ (\forall a,b,c \in L),\ a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$$
$$(D)^{\partial}\ (\forall p,q,r \in L),\ \mathrm{p} \vee (\mathrm{q} \wedge \mathrm{r}) = (\mathrm{p} \vee \mathrm{q}) \wedge (\mathrm{p} \vee \mathrm{r})\ .$$

4. Let L be a lattice. Then the following implications hold:
   (i) L is a Boolean algebra⟹L is relatively complemented.
   (ii) L is relatively complemented⟹L is sectionally complemented.
5. Use Karnaugh diagrams to simplify:

$$x_1x_2x_3 + x_2x_3x_4 + x_1'x_2x_4' + x_1'x_2x_3x_4' + x_1'x_2'x_4'.$$

B.A. (Programme)

Paper: Algebra (Semester II, CBCS)

Multiple Choice Questions

1. Rank of a zero matrix is
   (a) ϕ
   (b) 0
   (c) 1
2. A set consisting of a non-zero vector is
   (a) Linearly Independent
   (b) Linearly Dependent
3. Set of real numbers is a vector space over set of complex numbers
   (a) True
   (b) False
4. De Moivre's theorem: $(\cos\theta + \sin\theta)^n = \cos n\theta + \sin n\theta$ is true for n
   (a) n is real number
   (b) n is integer
   (c) n is natural number
5. A quadratic equation $\alpha x^2 + \beta x + \gamma = 0$ can have at the most
   (a) one solution
   (b) two solutions
   (c) three solutions

Short Answer Type Questions

1. Find the rank of the matrix $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 3 & 4 & 5 & 2 \\ 2 & 3 & 4 & 0 \end{bmatrix}$
2. Show that the set S= $\{\sin x\ ,\cos x\ \}$ is linearly Independent over set of real numbers.
3. Determine whether W = $\{a_0 + a_1 x \mid a_0, a_1 \in \mathbb{R}\}$ is a subspace of V = $\{a_0 + a_1 x + a_2 x^2 \mid a_0, a_1, a_2 \in \mathbb{R}\}$.
4. Use De Moivre's theorem to solve $z^5 - z = 0$.
5. Find all the roots of equation $x^4 + 2x^3 + x^2 = 0$.

Long Answer Type Questions

1. Show that the set {(1,0,-1) , (1,1,1) , (1,2,4)} is a basis for $\mathbb{R}^3$.

2. Solve the system of equations:

$$\begin{aligned} x - 4y + 7z &= 8 \\ 3x + 8y - 2z &= 6 \\ 7x - 8y + 26z &= 31 \end{aligned}$$

3. Define a Group and *Lagrange's* theorem for groups. Discuss the permutation group of n elements. Explain S_3 as a permutation group of three elements.

4. Define *Cayley-Hamilton* theorem. Find the inverse of the matrix

$$A = \begin{bmatrix} -1 & 2 & 0 \\ 1 & 1 & 0 \\ 2 & -1 & 2 \end{bmatrix}$$

Using Cayley-Hamilton theorem.

5. For what values of λ and μ do the following system of equations

$$\begin{aligned} x + y + z &= 6 \\ x + 2y + 3z &= 10 \text{ have} \\ x + 2y + \lambda z &= \mu \end{aligned}$$

(i) a unique solution

(ii) no solution

(iii) an infinitely number of solutions

Paper: Analysis (Semester IV, CBCS)

Multiple Choice Questions

1. The derived set of the set of the natural numbers is:

   (a) $\mathbb{N}$

   (b) $\mathbb{Z}$

   (c) $\emptyset$

   (d) $\mathbb{R}$

2. A non-empty subset of real numbers which is bounded below has:

   (a) Infimum

   (b) Both infimum and supremum

   (c) Supremum

   (d) Neither infimum nor supremum

3. Which of the following set is a neighbourhood of each of its points?

   (a) $\mathbb{Q}$

   (b) $\mathbb{N}$

   (c) $\mathbb{Z}$

   (d)]0,1[

4. Series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if:

   (a) $p < 1$

   (b) $p > 1$

   (c) $p = 1$

   (d) $p \leq 1$

5. For Riemann integrability on $[a, b]$, condition of continuity is

   (a) Necessary

   (b) Sufficient

   (c) Necessary and Sufficient

   (d) None of these

6. The sum of the series $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \cdots$ equals

   (a) 1

   (b) 0

   (c) 2

   (d) -1

7. For the function f defined on [0,3] as $f(x) = [x]$ (greatest integer function), the value of $\int_0^3 f(x)dx$ equals

   (a) 3

   (b) 2

   (c) 0

   (d) 1

8. Let f be a continuous function on $[a, b]$. Then

   (a) f is unbounded

   (b) f is monotonically increasing

   (c) f is constant on $[a, b]$

   (d) f is Riemann integrable

9. Let $f : [0,2] \to \mathbb{R}$ be defined as $f(x) = x[x] \ \forall \ x \in [0,2]$. Then

   (a) $\int_0^2 f(x)\, dx$ exists

   (b) $\int_0^2 f(x)\, dx$ =1

   (c) $\int_0^2 f(x)\, dx$ does not exist

   (d) None of the above

10. Let f be discontinuous at finitely many points of $[a, b]$. Then

   (a) f is Riemann integrable

(b) f is uniformly continuous

(c) f is differentiable

(d) None of the above

Short Answer Type Questions

1. Find the supremum and infimum of the set

$$S = \{1, 1+\frac{1}{2},\ 1+\frac{1}{2}+\frac{1}{2^2}, \ldots,\ 1+\frac{1}{2}+\frac{1}{2^2}+\cdots+\frac{1}{2^{n-1}}, \ldots\}$$

2. Prove that the set of integers $\mathbb{Z}$ has no limit point.

3. Show that the sequence (a_n), where $a_n = 1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots+\frac{1}{n}$ does not converge.

4. Show that the series $\sum_{n=1}^{\infty} e^{-n^2}$ converges.

5. Show that the function f defined on [0,1] by $f(x) = \begin{cases} 1, & if\ x \in \mathbb{Q} \\ 0, & if\ x \notin \mathbb{Q} \end{cases}$ is not Riemann integrable.

6. Find upper sum of the function $f(x) = k\ \forall\ x \in [0,2]$, where $k > 0$ is a constant.

Long Answer Type Questions

1. Prove that every convergent sequence is bounded. Justify with an example that the converse need not be true.

2. Prove that the intersection of two open sets is open. Give an example to show that the intersection of an arbitrary family of open sets need not be open.

3. Let $\sum_{n=1}^{\infty} u_n$ and $\sum_{n=1}^{\infty} v_n$ be two positive term series such that $\lim_{n\to\infty} \frac{u_n}{v_n}$ = c (c is finite and non-zero real number), then prove that $\sum_{n=1}^{\infty} u_n$ and $\sum_{n=1}^{\infty} v_n$ converge or diverge together.

4. Prove that a continuous function defined on a closed bounded interval is uniformly continuous. Justify with an example that the condition of the interval being closed cannot be relaxed.

5. Prove that if f is monotonic on $[a, b]$, then it is Riemann integrable on $[a, b]$.

6. A bounded function f is integrable on $[a, b]$ iff for every $\varepsilon > 0$ there exists a partition P of $[a, b]$ such that $U(P, f) - L(P, f) < \varepsilon$.

Paper: Numerical Analysis and Statistics (Semester VI)

Multiple Choice Questions

1. A coefficient of correlation is computed to be -0.95 means that

(a) The relationship between two variables is weak

(b) The relationship between two variables is strong and positive

(c) The relationship between two variables is strong and but negative

(d) Correlation coefficient cannot have this value

2. If $\beta_1 = 9$ and $\beta_2 = 11$, then coefficient of skewness is
   (a) 0.589
   (b) 0.689
   (c) 0.489
   (d) 0.889
3. If *Y=5X + 10* and *X* is *N(10,25)*, then mean of *Y* is
   (a) 50
   (b) 135
   (c) 60
   (d) 70
4. We wish to solve $x^2 - 2 = 0$ by Newton Raphson technique. If initial guess is $x_0 = 1.0$, subsequent estimate of x (i.e. x_1) will be
   (a) 1.414
   (b) 1.5
   (c) 2.0
   (d) None of these
5. The value of $\int_{0.2}^{2.2} e^x \, dx$ by using 2-segment Simpson's $1/3$ rule most nearly is
   (a) 7.8036
   (b) 7.8423
   (c) 8.4433
   (d) 10.246

Short Answer Type Questions

1. Calculate the nth divided difference of $1/x$, based on the points $x_0, x_1, x_2, \dots, x_n$.
2. Given that $f(2) = 4$, $f(2.5) = 5.5$, find the linear interpolating polynomial using Lagrange interpolation.
3. Consider the system of equations

$$\begin{bmatrix} 1 & -a \\ -a & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

   where a is a real constant.

   For what values of a, the Jacobi and Gauss-Seidel method converge.
4. Determine Mean and Variance of Poisson distribution.
5. Determine the Moment generating function of Continuous Uniform distribution.
6. Let X have the pdf $f(x) = 2(1 - x), 0 < x < 1$, and zero elsewhere. Find the Mean and $E(6X + 3X^2)$.
7. Perform five iterations of the Bisection Method to find a real root of the equation

$x^3 - x - 11 = 0$ in the interval $(2, 3)$.

Long Answer Type Questions

1. Perform five iterations of the Secant Method to find a real root of the equation $x^3 - 5x + 1 = 0$ in the interval $(0, 1)$.
2. Show that Newton-Raphson Method has second order of convergence.
3. Construct the divided difference table for the data:

x:	0.5	1.5	3.0	5.0	6.5	8.0
$f(x)$:	1.625	5.875	31.0	131.0	282.125	521.0

Hence, find the interpolating polynomial and an approximation to the value of $f(7)$.

4. Calculate the Coefficient of correlation from the following data.

X	12	9	8	10	11	13	7
Y	14	8	6	9	11	12	3

5. Show that Poisson distribution is a limiting case of Binomial distribution, when $n \to \infty, \theta \to 0$ such that $n\theta = \lambda$ (constant).
6. Perform three iterations of Jacobi Method to solve the following system of equations:

$$4x_1 + x_2 + x_3 = 2$$
$$x_1 + 5x_2 + 2x_3 = -6$$
$$x_1 + 2x_2 + 3x_3 = -4$$

taking the initial approximation $x^{(0)} = [0.5, -0.5, -0.5]$.

7. Solve the system of equations

$$\begin{bmatrix} 2 & 1 & 1 & -2 \\ 4 & 0 & 2 & 1 \\ 3 & 2 & 2 & 0 \\ 1 & 3 & 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -10 \\ 8 \\ 8 \\ -5 \end{bmatrix}$$

using the Gauss elimination method with partial pivoting.

Paper: Generic 2 - Linear Algebra

Multiple Choice Questions

1. The rank of the matrix

$$A = \begin{bmatrix} 3 & -1 & 2 \\ 2 & 1 & 3 \\ 7 & 1 & 8 \end{bmatrix} \quad \text{is}$$

(a) 0

(b) 1

(c) 2

(d) 3

2. The characteristic polynomial of the matrix

$$A = \begin{bmatrix} 12 & -51 \\ 2 & -11 \end{bmatrix} \quad \text{is}$$

(a) $x^2 - x - 30$

(b) $x^2 - x + 30$

(c) $x^2 + x + 30$

(d) $x^2 + x - 30$

3. The eigen values of the matrix

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & -3 \\ 0 & 0 & -5 \end{bmatrix} \text{ are}$$

(a) $1, 2, -5$

(b) $-1, -2, 5$

(c) $1, 2, -3$

(d) $1, 2, 5$

4. Let S be the subset of a vector space V. Then which of the following statement is false?

(a) $span(S)$ is defined only if S is a finite subset

(b) $S \subseteq span(S)$

(c) $span(S)$ is a subspace of V

(d) $span(S)$ is the smallest subspace of V containing S

5. Let $S = \{ [7,1,2,0], [8,0,1,-1], [1,0,0,-2] \}$. Then which of the following statement is true?

(a) S forms basis for R^4

(b) S is linearly independent

(c) S spans R^4

(d) S is linearly dependent

6. Which of the following subset of $\mathbb{R}^2$is a subspace of $\mathbb{R}^2$?

(a) The set of vectors of the form $[1, a]$

(b) The set of vectors of the form $[a, 2a]$

(c) The set of vectors of the form $[a, b]$, where $|a| = |b|$

(d) The set of vectors having a zero in at least one coordinate

7. Let x and y be vectors in $\mathbb{R}^n$. Then which of the following statement is false?

(a) x and y are orthogonal if and only if $x.y = 0$

(b) $||x - y|| \leq ||x|| - ||y||$

(c) $||x + y|| \leq ||x\,|| + ||y||$

(d) $\Big|\,||x|| \; - ||y||\,\Big| \leq \; ||x + y||$

Short Answer Type Questions

1. Show that the set $\mathbb{R}^2$, with the usual scalar multiplication but with vector addition replaced by

$$[x, y] \oplus [w, z] = [x + w, 0],$$

does not form a vector space.

2. If x and y are vectors in $\mathbb{R}^n$, then prove that

$$||x + y|| \;\leq ||x|| \;+ ||y||$$

3. Let V and W be vector spaces, and let $L\colon V \rightarrow W$ be a linear transformation. Let 0_v be the zero vector in V and 0_w be the zero vector in W. Then

$$L(0_v) \;=\; 0_w$$

$$L(-v) = \;-L(v) \quad for\; v \epsilon V.$$

4. Find the angle between the vectors

$$x = [8, -20,4] \text{ and } y = [6, -15,3].$$

5. Show that the mapping $f\colon \mathbb{R}^3 \rightarrow \mathbb{R}^3$ given by

$$f([x, y, z]) = [-x, y, z] \text{ is a linear operator.}$$

Long Answer Type Questions

1. Show that the Linear transformation $L\colon \mathbb{R}^2 \rightarrow \mathbb{R}^2$ given by

$$L\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} -3 & 4 \\ -6 & 9 \\ 7 & -8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

is one to one but not onto.

2. Let $L\colon \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be the linear operator given by

$$L\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 1 & -1 & 5 \\ -2 & 3 & -13 \\ 3 & -3 & 15 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Find a basis for $ker(L)$ and a basis for $range(L)$. Also verify the dimension theorem.

3. Solve the following system of linear equations using Gauss-Jordan method:

$$x_1 \;+\; 2x_2 \;+ 3x_3 \;=\; 6$$

$$2x_1 \;- 3x_2 \;+ 2x_3 \;= 14$$

$$3x_1 \;+\; x_2 - x_3 \;= -2.$$

4. State and prove *Cauchy -Schwartz Inequality*.

5. Use the simplified span method to find a simplified general form for all the vectors in $span(S)$, where $S = \{\, [1, -1,1], [2, -3,3], [0,1, -1] \,\}$ is a subset of $\mathbb{R}^3$.

Paper: Generic 4 - Numerical Methods

Multiple Choice Questions

1. The interpolating polynomial of highest degree which corresponds the functional values $f(-1)=9, f(0)=5, f(2)=3, f(5)=15,$ is

   (a) x^3+x^2+2x+5

   (b) x^2-3x+5

   (c) $x^4+4x^3+5x^2+5$

   (d) $x+5$

2. The Runge-Kutta method of order four is used to solve the differential equation

   $\frac{dy}{dx}=f(x),\ y(0)=0$ with step size h. the solution at $x=h$ is given by

   (a) $y(h)=\frac{h}{6}\left[f(0)+4f(\frac{h}{2})+f(h)\right]$

   (b) $y(h)=\frac{h}{6}\left[f(0)+2f(\frac{h}{2})+f(h)\right]$

   (c) $y(h)=\frac{h}{6}\left[f(0)+f(h)\right]$

   (d) $y(h)=\frac{h}{6}\left[2f(0)+f(\frac{h}{2})+2f(h)\right]$

3. The values of the constants α,β,x_1 for which the quadratic formula

   $$\int_0^1 f(x)dx=\alpha f(0)+\beta f(x_1)$$

   is exact for polynomials of degree as high as possible, are

   (a) $\alpha=\frac{2}{3},\beta=\frac{1}{4},x_1=\frac{3}{4}$

   (b) $\alpha=\frac{3}{4},\beta=\frac{1}{4},x_1=\frac{2}{3}$

   (c) $\alpha=\frac{1}{4},\beta=\frac{3}{4},x_1=\frac{2}{3}$

   (d) $\alpha=\frac{2}{3},\beta=\frac{3}{4},x_1=\frac{1}{4}$

4. On evaluating $\int_1^2\int_1^2 \frac{1}{(x+y)}dxdy$ numerically by trapezoidal rule one would get the value

(a) $\frac{17}{48}$

(b) $\frac{11}{48}$

(c) $\frac{21}{48}$

(d) $\frac{17}{52}$

5. Let $f(x)$ be an equation such that $f(a)f(b) > 0$ for two real numbers a and b. then

(a) At least one root of $f(x) = 0$ lies in (a, b)

(b) No root lies in (a, b)

(c) Either no root or an even number of roots lie in (a, b)

(d) None of these

Short Answer Type Question

1. Define round-off error and truncation error.
2. Give geometrical interpretation of :

a. Secant method

(iii) Newton Raphson method

3. Find the minimum number of iterations required to attain an accuracy of 0.001 in the interval [1, 2] using bisection method.
4. Using the data given in the table

X	1	2	3	4
f(x)	2	4	8	16

Estimate the second derivative at x=3.

5. Given f(2)=4, f(2.5)=5.5, find the linear interpolating polynomial using Lagrange interpolation. Hence, find an approximate value of f(2.2).

Long Answer Type Questions

1. Perform five iterations to find a positive real root of

$$x^4 - 0.65=0$$

using Regula Falsi method.

2. Construct the interpolating polynomial that fits the data

x	0	0.1	0.2	0.3	0.4	0.5
f(x)	-1.5	-1.27	-0.98	-0.63	-0.22	0.25

using the Gregory-Newton forward or backward difference interpolation. Hence, or otherwise estimate the values of $f(x)$ at $x = 0.15$, 0.25 and 0.45.

3. Using Gauss-Seidel method and the first iteration as (0, 0, 0), calculate the next three iterations for the solution of the system of equations :

$$
\begin{aligned}
5x - y + z &= 10 \\
2x + 8y - z &= 11 \\
-x + y + 4z &= 3.
\end{aligned}
$$

4. Obtain the cubic spline approximation for the function defined by the data

x	0	1	2	3
f(x)	1	2	33	244

with M(0)=0, M(3)=3. Hence find the estimate of $f(2.5)$.

5. Find the remainder of the Simpson three-eighth rule

$$\int_{x_0}^{x_1} f(x)dx = \frac{3h}{8}\left[f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3)\right]$$

for equally spaced point $x_i = x_0 + ih, i = 1, 2, 3$. Use this rule to approximate the value of the integral $I = \int_0^1 \frac{dx}{1+x}$

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